Estimation of Stated Preferences from Incomplete Rankings*

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Abstract

In this paper a cascading choice model based on Luce's Choice Axiom is extended to situations with incomplete rank-order data where only the ranks of some of the best and worst alternatives are known. The exact ranking of alternatives in this intermediate range are not known. The maximum likelihood estimator for this behavioral model exists, and is computationally feasible. One of the less desirable features of this type of behavioral model is the lack of reversibility of the rankings. This may limit the applicability of such a model despite its computational ease.

1 Introduction

There is increasing interest in stated preference data both for transportation demand analysis and valuation of non-market goods. In one response format for stated preference analysis are all the alternatives in the choice set ranked in order of decreasing preference. The observed ranking data is often assumed to be generated by a probabilistic process which satisfies Luce's Choice Axiom. The Luce and Suppes' cascading choice theorem states that a ranking of alternatives is equivalent to a sequence of independent choice situations. That is, the alternative given rank one is the choice when all alternatives are available, the alternative given rank two is the choice when all alternatives except the alternative given rank one is available, etc. This leads typically to the empirical specification of a multinominal logit model based either on complete ranking of all alternatives in the choice set or a partial ranking involving only some of the best alternatives.

In this paper the analysis of choice and ranking is extended to situations with censored rank-order data where the ranks of some of the best and worst alternatives are known. The exact ranking of alternatives in the range between the groups of best and worst alternatives are not known.

The advantage in using censored rank-order data is in its efficiency relative to choice and partial ranking data. Complete rank-order data has proven difficult to obtain in many experimental settings when the choice set contains more than a few elements, and the literature reports empirical evidence that the stability of ranking information decreases with increasing rank order. Censored rank-order data are viewed as easier to elicit experimentally. Thus it is of iterest to gain insight into the theoretical properties of such incomplete rankings.

The purpose of this paper is to

1. give an overview of some of the existing results in the literature pertaining to choice and ranking models, and

2. extend this analysis to situations with incomplete rankings.

The paper starts with a brief review of stated preference techniques, and especially how such techniques are used in the non-market valuation. The following section reviews a random utility model for choice analysis based in the Thurstone-Luce tradition and states several results. The relationship between rankings and choice is given particular attention. Estimation the preference parameters are easily accomplished with maximum likelihood techniques applied to a multinominal logit type of model. Some problems with this approach to rankings and choice are discussed in the last section, and some avenues for avoiding these problems are indicated.

2 Stated Preference Experiments

There are a number of methods available for determining preferences through experiments. Especially in psychology does there exists a large research tradition on this topic[1]. There is also increasing interest in and use of general experimental methods in economics (Fridstrøm 1992, Davis and Holt 1993).

However, the motivation for this research is in the estimation of preferences for complex environmental goods and the valuation of multidimensional changes in environmental amenities and services. Thus the intentions are not to provide a comprehensive review of stated preference experiments, but rather to indicate how they are applied in non-market valuation studies.

In a simple choice experiment are the participants asked to select among two or more alternatives according to some specified criterion, and in a ranking experiment is he/she asked to rank order the alternatives, again according to some specified criterion. Ranking experiments can elicit the entire ranking or only a partial rank order.

2.1 Nonmarket valuation methods

Economists have devised a number of methods for assigning a value, or price, to those goods and services not routinely traded in fully functioning markets. The nonmarket valuation methods can be divided into[2]

[1] One important tradition starts with the work of Thurstone (1927), continuing on with Coombs (1964), Krantz, Luce, Suppes and Tversky (1971) and Suppes, Krantz, Luce and Tversky (1989).

[2] Bergland (1993) gives a brief overview of such methods. Pearce and Markandya (1989) and Bergstrom (1990) give systematic introductions to different nonmarket valuation techniques. See the volume by Braden and Kolstad (1991) for a state of the art review.

1. revealed preference methods, and
2. stated preference methods.

Revealed preference methods are widely used for valuation of many types of nonmarket goods. Important classes of methods are hedonic pricing models, household production function models and models base on preference interdependencies such as weak complementarity.

The dominant nonmarket valuation method based on stated preferences is the *contingent valuation* method. The (modern) roots of the method goes back to Randall, Ives and Eastman (1974) and Brookshire, Ives and Shulze (1976). The standard reference on the method is Mitchell and Carson (1989)[3].

The basic idea behind contingent valuation is to ask the participants in the valuation experiment more or less directly about the willingness-to-pay for a hypothetical. The elicited value is contingent upon the scenario specified in the experiment. The value elicitation component of the experiment can be

1. directe questions about maximum willingness-to-pay or minimal compensation,
2. referendum style yes/no-questions, or
3. iterative bidding games.

Of particular interest here is the referendum type contingent valuation method.

Alternative methods for stated preferences are contingent ranking (Rae 1983) and conjoint analysis (Green and Srinivasan 1978), although neither have hardly been used for nonmarket valuation. They will be briefly reviewed.

2.2 Contingent Ranking

Contingent ranking is an alternative method to contingent valuation proposed in the early eighties (Rae 1983). The method is implemented much in the same way as contingent valuation, thus exposing itself to much of the same criticisms levelled against contingent valuation.

However, the method differs from contingent valuation in that the respondent in the experiment is asked to rank a large number of alternatives with combinations of environmental goods and prices as compared to the two alternatives in the referendum format of contingent valuation. The complete ranking data is analyzed with the econometric technique of

[3]The method is further discussed by Brookshire and Crocker (1981), Cummings, Brookshire and Schulze (1986) and Fischhoff and Furby (1988).

Beggs, Cardell and Hausman (1981), and implicit attribute prices are then imputed from the parameter estimates.

The contingent ranking method has met with mixed responses (Smith and Desvousges 1986, Lareau and Rae 1989). The implementations of contingent ranking has typically involved the ranking of a very large number of alternatives which often appear similar to the respondent. The cognitive task of arriving at a complete ranking is often experienced as a very difficult and demanding task. The final statistical model of the rankings are often poor which results in questionable imputed prices.

2.3 Conjoint analysis

Conjoint analysis is a method widely used in marketing, although with strong roots in psychology and statistics (Luce and Tukey 1964, Kruskal 1965, Green and Srinivasan 1978). From the point of individual choice theory as used in economics, the theoretical foundation of conjoint analysis seems rather shaky (Madansky 1980, McFadden 1986, Bates 1988). There is a trend in conjoint analysis from reliance on pure statistical methods towards more behaviorally based models such as the multinominal logit model (Louviere 1988a).

The strength of conjoint analysis is in the explicit use of statistical experimental design techniques to explore a number of different attributes in a choice or ranking setting. However, this design feature need not be limited to the conjoint analysis, but could be linked with more general random utility models of choice behavior (Hensher 1982, Bates 1988, Louviere 1988b).

One feature of conjoint analysis is that one individual is faced with a large number of ranking tasks. Based on the collected data, some type of utility index model is estimated for *one* individual. In contingent valuation and ranking a large number of individuals are asked about their stated preferences, and a *representative* random utility model is then estimated for the relevant population.

3 Models of Choice and Ranking

Based upon the Bradely-Terry-Luce model (Bradley and Terry 1952, Luce 1959) of ranking data and individual choice it is possible to formulate *random utility* models of ranking and choice data (Block and Marschak 1960, Marschak 1960). Using Luce's choice theorem (Luce 1959) and the cascading choice theorem of Luce and Suppes (1965), ranking data can be transformed into choice data (Chapman and Staelin 1982). That is, the alternative given rank one is the choice when all alternatives are available, the alternative given rank two is the choice when all alternatives except the alternative given rank one is available, etc. The purpose of this section is to explore the use of the Bradely-Terry-Luce model to ranking data in more detail.

3.1 Notation

Let A be the universal set of alternatives, and suppose that an individual faces a finite set of choices, $C \subseteq A$. Let $P_C(c)$ denote the probability that c is chosen from an available set C of alternatives. If $S \subseteq C$, then $P_C(S)$ denotes the probability that the selected element lies in the subset S.

The function $P_C(\cdot)$ defines a standard probability measure[4] on the subsets of C for fixed choice set C, i.e.

Axiom 1 (Probability Measure)

1. *For all* $S \subset C$, $0 \leq P_C(S) \leq 1$.

2. $P_C(C) = 1$.

3. *If* $R, S \subset C$ *and* $R \cap S = \emptyset$, *then* $P_C(R \cup S) = P_C(T) + P_C(S)$.

The following result concerning summation of probabilities for mutually exclusive events is well-known from the theory of probabilities

$$P_C(S) = \sum_{s \in S} P_C(s). \tag{1}$$

The probability measure defined on C constitutes a structure of choice probabilities which is closed for finitie A. Such structures of choice probabilities can be analyzed either with a *strict utility model* or a *random utility model*.

Definition 1 *A closed structure of choice probabilities satisfies the* strict utility model *iff there exists a positive real-valued function* ψ *on* A *such that for all* $c \in C \subseteq A$,

$$P_C(c) = \frac{\psi(c)}{\sum_{s \in C} \psi(s)}. \tag{2}$$

The random utility model traces its historical roots back to the pioneering work by Thurstone (1927), and such models are also known as Thurstonian models in psychology.

Definition 2 *A closed structure of choice probabilities satisfies the* random utility model *iff there exists a collection* $U = \{u_a : a \in A\}$ *of jointly distributed random variables such that for all* $c \in C \subseteq A$,

$$P_C(c) = \Pr(u_c \geq u_s \quad \forall s \in C). \tag{3}$$

Thus the probability of observing a particular alternative c chosen is equal to the probability that this alternative has the greatest utility value. One approach to analyzing choice data is then with the help of order statitiscs (Critchlow, Fligner and Verducci 1991). There is, however, an important research tradition based upon a particular behavioral model.

[4] See for example Chung (1974) for an exposition of probability theory.

3.2 Luce's Choice Axiom

Some of the most important theoretical work concerning choice and ranking takes the Choice Axiom of Luce (1959) as its point of departure.

Axiom 2 (Choice Axiom)
A closed structure of choice probabilities, with $P_C(S) \neq 0$ *for all* $S \subseteq C$*, satisfies the* choice axiom *iff for all* $T \subseteq S \subseteq C$

$$P_C(T) = P_S(T)P_C(S). \tag{4}$$

The choice axiom asserts basically that the choice process leading to the selection of T (or an element of T) from the total set C of available alternatives can be decomposed into independent choices:

1. the choice of T from S, and
2. the choice of S from C.

There are a number of consequences of the Choice Axiom of which only a few will be repeated here[5]. The following theorem is often taken as an alternative statement of the choice axiom.

Theorem 1
The choice axiom implies that the following holds for any $s, c \in C$

$$\frac{P_{\{s,c\}}(s)}{P_{\{s,c\}}(c)} = \frac{P_C(s)}{P_C(c)}. \tag{5}$$

This result is known alternatively as the *constant ratio* rule or the *indipendence from irrelevant alternatives* property of choice. It is a difficult and restrictive feature of the choice axiom and one which is not always reasonable (McFadden 1981).

Theorem 2
The choice axiom implies that for A *and its subsets, that there exists a positive real-valued function* v *on* A*, which is unique up to multiplication by a positive constant, such that for every* $C \subset A$

$$P_C(c) = \frac{v(c)}{\sum_{s \in C} v(s)}. \tag{6}$$

Choice processes which satisfies the choice axiom will be rationalizable with some strict utility function.

As far as random utility model goes, it is commonly believed that the choice axiom (because of the independence from irrelevant alternatives) implies a multinominal type of random utility model. The following theorem shows that this is not exactly the case (Yellot 1977, Strauss 1979).

[5] See Luce and Suppes (1965) and Luce (1977).

Theorem 3
Let the random variables $(u_a; a \in A)$ *be independently distributed with a common distribution function* F*. Then the choice probabilities will satisfy the choice axiom if and only if* F *is double exponential.*

The additional assumption of independently distributed must be added in order to link the choice axiom up with the logit model. Of course, if the random utilities are assumed to be independently and identically distributed then the logit model is inevitable (Strauss 1979, McFadden 1973).

3.3 Decomposition

Thus far the concern has been with choice, but this modelling framework is extendable to the question of ranking. The exposition here follows to a large extent the review in Colonius (1984)

Consider the finite set of alternatives $C = \{c_1, c_2, \ldots, c_n\}$ where all the alternatives are indexed, and let $\mathcal{R}_C$ be the set of all possible permutations of the elements in C. One such permutation is $\rho \in \mathcal{R}_C$ where

$$\rho = (c_{i_1}, c_{i_2}, \ldots, c_{i_n})$$

is the permutation of the elements in C which gives alternative c_{i_1} rank number one, c_{i_2} rank number two, etc.. Let $\rho(j)$ denote the rank of alternative c_j, and thus $\rho^{-1}(i)$ is the index of the alternative with rank i.

Let $r(\rho)$ denote the probability of the rankorder ρ. We are now in position to define a probability measure on rankorders, i.e. the probability of a particular alternative having a specified position in the rankordering. The properties of such probability measures are given by the following result due to Block and Marschak (1960).

For a ranking on any set C the ranking probabilities are given by

$$\Pr(\rho(i) = j) = \sum_{\tau \in R_{i,j}} r(\tau) \qquad (7)$$

where $R_{i,j} = \{\rho \in \mathcal{R}_C : \rho(c_i) = j\}$

It is now possible to give a more precise characterization of relationship between choice and ranking in terms of decomposing the ranking to a sequence of choices. In particular is it taken to a requirement that choices and rankings are consistent with each other (Block and Marschak 1960).

Condition 1 *The* consistency condition *(between choice and ranking) states that a structure of choice probabilities satisfies a random utility model iff there exists a probability* *measure on rankings such that*

$$P_C(c_i) = \sum_{\rho \in R_i^*} r(\rho) \tag{8}$$

where $R_i^* = \{\rho \in \mathcal{R}_C : \rho(c_i) = 1\}$.

Condition 2 Complete decomposition *implies that the probability of a ranking of alternatives can be written as the probability of a sequence of choice*

$$r(c_i, c_j, \ldots, c_k, c_l) = P_{\{c_i, c_j, \ldots, c_k, c_l\}}(c_i) P_{\{c_j, \ldots, c_k, c_l\}}(c_j) \cdots P_{\{c_k, c_l\}}(c_k). \tag{9}$$

The following relationships between decomposition and the choice axiom are exists (Strauss 1979).

Theorem 4
For a random utility model

1. *complete decomposition implies the choice axiom,*
2. *the choice axiom implies decomposition for alternative sets with cardinality 3.*

Theorem 5
Let the random variables $(u_a; a \in A)$ *be independently distributed with a common distribution function* F*. Then the choice probabilities will satisfy complete decomposition if and only if* F *is double exponential.*

This is a version of the cascading choice theorem of Luce and Suppes (1965). Thus the choice axiom in itself is not strong enough to provide the necessary structure in order to make choice and ranking interchangeable. Additional assumptions about the distribution of the random errors are needed, and again i.i.d. assumptions are typical which immidiately leads to the multinominal logit model (Beggs et al. 1981, Chapman and Staelin 1982, Ben-Akiva, Morikawa and Shiroishi 1991). However, if we accept a common distribution function on the stochastic part of the random utility model, then our resulting model will satisfy the choice axiom, ensure consistency between choice and ranking, and permit a tractable statistical specification of the model.

3.4 Incomplete Rankings

A *partial* ranking of the k best alternatives among the n alternatives in a set $C \subseteq A$ is denoted $\rho_{k/n}$ where

$$\rho_{k/n} \in \{(c_{i_1}, \ldots, c_{i_n}) : c_{i_m} \in \{c \in C : k < \rho(c)\},\ m = k+1, \ldots, n\}$$

A *incomplete* ranking of the k best alternatives and the l worst alternatives among the n alternatives in a set $C \subseteq A$ is denoted $\rho_{(k,l)/n}$ where

$$\rho_{(k,l)/n} \in \{(c_{i_1}, \ldots, c_{i_n}) : c_{i_m} \in \{c \in C : k < \rho(c) < l\},\ m = k+1, \ldots, l-1\}$$

Under the assumptions of indipendently distributed random utilities from a double exponential distribution function will applications of probability calculus yield that the probability of a particular incomplete ranking $\rho_{(k,l)/n}$ is

$$\Pr(\rho_{(k,l)/n}) = \left(\prod_{j=1}^{k} P_{\{c_{i_j},\ldots,c_{i_n}\}}(c_j)\right)\left(\prod_{j=k+1}^{l-1} \sum_{S \in \mathcal{S}(|,\updownarrow)} P_S(c_j)\right)\left(\prod_{j=l}^{n} P_{\{c_{i_j},\ldots,c_{i_n}\}}(c_j)\right) \tag{10}$$

where

$$\mathcal{S}(|,\updownarrow) = \Big\{ (\rfloor_{\rangle_|}, \ldots, \rfloor_{\rangle_\backslash}) : \rfloor_{\rangle_\Updownarrow} \in \{\rfloor \in \mathcal{C} : | \leq \rho(\rfloor) < \updownarrow\},\ \Updownarrow = |, \ldots, \updownarrow - \infty \Big\}$$

The only difference between this probability for an incomplete rankorder and that for a multinominal logit model is in the middle term which takes into account the different permutations of the choice set consistent with the observed ranking.

As the incomplete ranking will yield a multinominal logit model it is a rather straightforward task to obtain the maximum likelihood estimates of the parameters of such choice model using standard nonlinear optimization methods (Dennis and Schnabel 1983).

3.5 Probabilistic choice formulation

The quantity rationing model of individual behavior provides the behavioral model necessary for linking the described alternatives with the choice data. Let individual t be faced with the choice set C^t which is a particular block of the fractional factorial design randomly assigned to individual t. Let the utility level associated with alternative s be denoted v_s.

The random utility associated with alternative s is

$$u_s = v_s + \epsilon_s$$

where ϵ_s is some unobserved stochastic term. The choice probability for alternative $c^* \in C^t$ is

$$\begin{aligned} P_{C^t}(c^*) &= \Pr\big\{u_{c^*} > u_s \quad \forall s \in C^t \setminus \{c^*\}\big\} \\ &= \Pr\big\{v_{c^*} + \epsilon_{c^*} > v_s + \epsilon_s \quad \forall s \in C^t \setminus \{c^*\}\big\}. \end{aligned} \tag{11}$$

Estimation of preference parameters in this model proceeds by using common maximum likelihood techniques for probabilistic choice models (Maddala 1983, Ben-Akiva and Lerman 1985), and expanding the likelihood function to incorporate the structure of the incomplete contingent ranking data from equation 10. Although this is not possible to do with currently available statistical packages, there exists a number of general nonlinear optimization routines (Dennis and Schnabel 1983) which can tailored for the current estimation problem.

3.6 Measures of value

The probabilistic choice modeling framework estimates the restricted indirect utility function. Thus, information is available about the virtual prices for different environmental characteristics through Gorman's Identity, or of the Hicksian welfare change measures. This information can be calculated directly from the estimated parameters.

4 Discussion

The incomplete ranking method is intended to be combined with contingent ranking to offer an alternative technique for determining preferences for complex environmental goods. Contingent valuation with referendum data can be used for this purpose, but the size of the required experiment turns out to be enormous (Hoehn 1991). Thus a modified contingent ranking approach seems in order.

In terms of ranking there are both theoretical and empirical evidence supporting the view that the link between choice and ranking breaks down as the rankorder is traversed (Chapman and Staelin 1982, Ben-Akiva et al. 1991). Thus a complete ranking of many alternatives may not be the best implementation of ranking experiments, but rather one should use an incomplete ranking with the few best alternatives and, possibly, the worst alternatives. The discussion here indicates that this is a feasible approach.

There is however a major obstacle to using the worst alternatives in a ranking experiments, and that is the reversibility paradox for ranking decompositions. Block and Marschak (1960) showed the surprising result that a ranking of three alternatives in terms of the best would only yield the same ranking in terms of the worst in the special case of indifference between the alternatives. There is by now a large literature on this topic (Luce and Suppes 1965, Thorson and Stever 1974, Colonius 1984, Critchlow et al. 1991), and the work by Yellot (1980) is particular pertinent. The result that forward and backward ranking should yield different results is counter-intuitive and quite damaging to this type of choice modelling which considers incomplete rankings.

Two alternative routes exists for dealing with the reversability paradox. As noted by Yellot (1980) the paradox is rooted in the fact that the double exponential distribution is asymmetric. Thus one approach is to explore alternative distributions which is consistent with some behavioral model, preferably the random utility model, and which yields decomposable rankings. An additional benefit of this approach is the potential to avoid the problem of independence from irrelevant alternatives. On the other hand is it very seldom computationally feasible to estimate anything but the multinominal logit model.

Another alternative is to design the experiment eliciting incomplete rankorders such as to force a uni-directional preference ordering throughout the experiment. Given the evidence about the heuristics in use when solving cognitive problems (Eysenck and Keane 1990) it will be difficult and costly to ensure that the assessment technique employed in an experimental setting is monotone from best to worst or vice versa.

The potential in contingent ranking based on incomplete rankings is still large, and further research into the theoretical and statistical properties of such a model is warranted.

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